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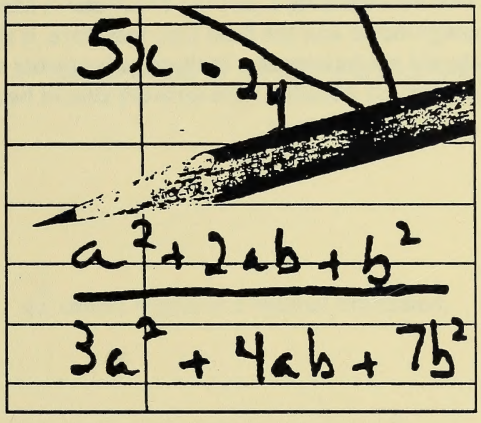
MATHEMATICS 33

Polynomials and Rational Expressions



Unit 2

Learning Facilitator's Manual

Distance Learning

Alberta EDUCATION

Note

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Mathematics 33
Learning Facilitator's Manual
Unit 2
Polynomials and Rational Expressions
Alberta Distance Learning Centre
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Topic 1: Operations with Rational Expressions

(5)

1. Identify the rational expressions. If the expression is a rational expression, write an **R** beside it. If the expression is not a rational expression, write an **N** beside it.

a. $\frac{1}{3}$ **R**

b. x^{-3} **R** x^{-3} can be written as $\frac{1}{x^3}$, and $\frac{1}{x^3}$ is a rational expression.

c. $\frac{7x+2}{x^2-1}$ **R**

d. $2\sqrt{x}+3$ **N** $\sqrt{x}$ cannot appear in a rational expression.

e. $x-\frac{1}{x}+4$ **R**

2. Simplify each of the following.

(1)

a. $\frac{14x^2y^5}{21x^3y^4}$

$$\frac{14x^2y^5}{21x^3y^4}=\frac{(2)(\cancel{7})(\cancel{x})(\cancel{x})(x)(\cancel{y})(\cancel{y})(\cancel{y})(\cancel{y})(y)}{(3)(\cancel{7})(\cancel{x})(\cancel{x})(\cancel{x})(x)(\cancel{y})(\cancel{y})(\cancel{y})(\cancel{y})}$$

$$=\frac{2y}{3x}$$

or $$\frac{14x^2y^5}{21x^3y^4}=\frac{\cancel{14}^{2}}{\cancel{21}_{3}}\times\frac{\cancel{x^2}^{1}}{\cancel{x^3}_{x}}\times\frac{\cancel{y^5}^{y}}{\cancel{y^4}_{1}}$$

$$=\frac{2\times1\times y}{3\times x\times1}$$

$$=\frac{2y}{3x}$$

(1) b. $\dfrac{6x^2+2x}{6x^3-7x^2-3x}=\dfrac{2x(3x+1)}{x(6x^2-7x-3)}$

$$=\frac{2\overset{1}{\cancel{x}}\overset{1}{\cancel{(3x+1)}}}{\underset{1}{\cancel{x}}\underset{1}{\cancel{(3x+1)}}(2x-3)}$$

$$=\frac{2}{2x-3}$$

(1) c. $\dfrac{2x^2+2x-4}{2x^2-8}=\dfrac{2(x^2+x-2)}{2(x^2-4)}$

$$=\frac{\overset{1}{\cancel{2}}\overset{1}{\cancel{(x+2)}}(x-1)}{\underset{1}{\cancel{2}}\underset{1}{\cancel{(x+2)}}(x-2)}$$

$$=\frac{x-1}{x-2}$$

(1) d. $\dfrac{3x^2-2x-5}{9x^2-25}=\dfrac{\overset{1}{\cancel{(3x-5)}}(x+1)}{\underset{1}{\cancel{(3x-5)}}(3x+5)}$

$$=\frac{x+1}{3x+5}$$

(3) 3. A bag of candy has a mass of $(15x^3-2x^2-x)$g. The bag contains $(3x-1)$ candies. Write a rational expression, in simplest form, that represents the average mass of each piece of candy.

$$\frac{15x^3-2x^2-x}{3x-1}=\frac{x(15x^2-2x-1)}{3x-1}$$

$$=\frac{x\overset{1}{\cancel{(3x-1)}}(5x+1)}{\underset{1}{\cancel{(3x-1)}}}$$

$$=x(5x+1)$$

Each piece of candy has a mass of $x(5x+1)$g.

4. Determine the nonpermissible values for each of the following rational expressions.

To find the nonpermissible values for a rational expression, you let the denominator equal zero and then solve for the variable in the denominator only.

(1) a. $\frac{3}{x}$

$x \neq 0$, or the nonpermissible value for x is 0.

(1) b. $\frac{y-2}{xy}$

If $xy = 0$, then $x = 0$, $y = 0$.
Therefore, $x \neq 0$, $y \neq 0$, or the nonpermissible value for y is 0 and for x is 0.

(1) c. $\frac{x-3}{5x-4}$

$$5x-4=0$$
$$5x=4$$
$$x=\frac{4}{5}$$

Therefore, $x \neq \frac{4}{5}$, or the nonpermissible value is $\frac{4}{5}$.

(2) d. $\frac{x-5}{x^2-5x-24}$

$$x^2-5x-24=0$$
$$(x-8)(x+3)=0$$
$$x-8=0 \text{ or } x+3=0$$
$$x=8 \qquad x=-3$$

Therefore, $x \neq 8$, -3, or the nonpermissible values are 8 and -3.

(2) e. $\frac{x^2-5x-6}{(x^2-9)(2x^2-5x-7)}$

$$(x^2-9)(2x^2-5x-7)=0$$
$$(x+3)(x-3)(2x-7)(x+1)=0$$
$$x+3=0,\ x-3=0,\ 2x-7=0, \text{ or } x+1=0$$
$$x=-3 \quad x=3 \quad x=\frac{7}{2} \quad x=-1$$

Therefore, $x \neq 3$, -3, $\frac{7}{2}$, -1, or the nonpermissible values are 3, -3, $\frac{7}{2}$, and -1.

(2) 5. Why is division of a natural number by zero undefined?

The quotient is so large that it cannot be found and named.

(1) 6. If $(x)(y)=0$, then what can you say about the possible values for x or y?

If $(x)(y)=0$, then $x=0$ or $y=0$.

(1) 7. Simplify the expression $\frac{a-b}{b-a}$.

$$\frac{a-b}{b-a}=\frac{a-b}{-1(-b+a)}$$

$$=\frac{(-1)(\cancel{a-b})}{(\cancel{a-b})}$$

$$=-1$$

(1) 8. Simplify the expression $\frac{10x^{50}y^{70}}{5x^{48}y^{73}}$.

$$\frac{10x^{50}y^{70}}{5x^{48}y^{73}}=\frac{\cancel{10}^{2}}{\cancel{5}_{1}}\times\frac{\cancel{x^{50}}^{x^2}}{\cancel{x^{48}}_{1}}\times\frac{\cancel{y^{70}}^{1}}{\cancel{y^{73}}_{y^3}}$$

$$=\frac{2\times x^2\times 1}{1\times 1\times y^3}$$

$$=\frac{2x^2}{y^3}$$

(4) 9. Multiply and simplify.

$$\frac{a^2-2a}{a^2-4}\times\frac{a^2+2a}{a^2}$$

$$\frac{\overset{1}{a}(a-2)}{(a+2)(a-2)}\times\frac{\overset{1}{a}(a+2)}{(a)(a)}=1$$

(3) 10. Divide and simplify.

$$\frac{3t^2}{-2y}\div\frac{6t^3}{4y^2}$$

$$\frac{3t^2}{-2y}\div\frac{6t^3}{4y^2}=\frac{3t^2}{-2y}\times\frac{4y^2}{6t^3}$$

$$=\frac{\overset{1}{3}\times\overset{2}{4}}{-2\times 6}\times\frac{\overset{1}{t^2}}{t^3}\times\frac{\overset{y}{y^2}}{y}$$

$$=\frac{\overset{1}{2}}{-2}\times\frac{1}{t}\times\frac{y}{1}$$

$$=\frac{-y}{t}$$

or $$\frac{3t^2}{-2y}\times\frac{4y^2}{6t^3}=\frac{(3)(t)(t)}{(-2)(y)}\times\frac{(2)(2)(y)(y)}{(2)(3)(t)(t)(t)}$$

$$=\frac{1}{-1}\times\frac{y}{t}$$

$$=\frac{-y}{t}$$

(4) 11. Divide and simplify.

$$\frac{x-3}{2-x} \div \frac{2x-6}{x-2}$$

$$\frac{x-3}{2-x} \times \frac{x-2}{2x-6} = \frac{\overset{1}{\cancel{(x-3)}}}{-1\underset{1}{\cancel{(-2+x)}}} \times \frac{\overset{1}{\cancel{(x-2)}}}{2\underset{1}{\cancel{(x-3)}}}$$

$$= \frac{1}{-1} \times \frac{1}{2}$$

$$= \frac{-1}{2}$$

(5) 12. Multiply and simplify.

$$\frac{11ab^3}{5a^2b^4} \times \frac{10a^3b}{22ab}$$

$$\frac{11ab^3}{5a^2b^4} \times \frac{10a^3b}{22ab} = \frac{\overset{1}{\cancel{11}} \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}} \times \underset{2}{\cancel{22}}} \times \frac{\overset{a}{\cancel{a^4}}}{\underset{1}{\cancel{a^3}}} \times \frac{\overset{1}{\cancel{b^4}}}{\underset{b}{\cancel{b^5}}}$$

$$= \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}} \times \frac{a}{1} \times \frac{1}{b}$$

$$= \frac{a}{b}$$

(5) 13. Multiply and simplify.

$$\frac{5x^2-45}{x^2-6x-7} \times \frac{x^2-1}{3x^2+11x+6}$$

$$\frac{5\overset{1}{\cancel{(x+3)}}(x-3)}{(x-7)\underset{1}{\cancel{(x+1)}}} \times \frac{\overset{1}{\cancel{(x+1)}}(x-1)}{(3x+2)\underset{1}{\cancel{(x+3)}}} = \frac{5(x-3)(x-1)}{(x-7)(3x+2)}$$

(5) 14. Divide and simplify.

$$\frac{3x+12}{6x+18} \div \frac{x^2+4x-12}{x^2+9x+18}$$

$$\frac{3x+12}{6x+18} \times \frac{x^2+9x+18}{x^2+4x-12} = \frac{\overset{1}{\cancel{3}}(x+4)}{\underset{2}{\cancel{6}}(\underset{1}{\cancel{x+3}})} \times \frac{(\overset{1}{\cancel{x+6}})(\overset{1}{\cancel{x+3}})}{(\underset{1}{\cancel{x+6}})(x-2)}$$

$$= \frac{x+4}{2} \times \frac{1}{x-2}$$

$$= \frac{x+4}{2(x-2)}$$

(1) 15. Before two rational expressions can be added, they must have the same **denominator**.

16. Add or subtract, as indicated, and simplify the following.

(2) a. $\frac{2x+3}{x-2} - \frac{x+5}{x-2}$

$$\frac{2x+3-(x+5)}{x-2} = \frac{2x+3-x-5}{x-2}$$

$$= \frac{x-2}{x-2}$$

$$= 1$$

(2) b. $\frac{x}{x+y} + \frac{y}{x+y}$

$$\frac{x+y}{x+y} = 1$$

17. Find the L.C.D. for each rational expression.

(2) a. $\frac{2}{3x^2} - \frac{5}{2x}$

$3x^2 = (3)(x)(x)$

$2x = (2)(x)$

$\text{L.C.D.} = (3)(2)(x)(x)$

$= 6x^2$

(2) b. $\frac{5}{4x^2y} + \frac{10}{6xy^3}$

$4x^2y = (2)(2)(x)(x)(y)$

$6xy^3 = (2)(3)(x)(y)(y)(y)$

$\text{L.C.D.} = (2)(2)(3)(x)(x)(y)(y)(y)$

$= 12x^2y^3$

(2) c. $\frac{2}{x^2 - 3x} + \frac{5}{x^2 - 9}$

$x^2 - 3x = x(x-3)$

$x^2 - 9 = (x+3)(x-3)$

$\text{L.C.D.} = x(x-3)(x+3)$

18. Add or subtract, as indicated, and simplify the following.

(3) a. $\dfrac{3x+1}{x-1}+\dfrac{2x+2}{1-x}$

The denominators are negatives of each other.

$$\frac{3x+1}{x-1}+\frac{(2x+2)}{(1-x)}\frac{(-1)}{(-1)}=\frac{3x+1}{x-1}+\frac{(-2x-2)}{-1+x}$$

$$=\frac{3x+1}{x-1}+\frac{(-2x-2)}{x-1}$$

$$=\frac{3x+1-2x-2}{x-1}$$

$$=\frac{\overset{1}{\cancel{(x-1)}}}{\underset{1}{\cancel{(x-1)}}}$$

$$=1$$

(3) b. $\dfrac{2x+3}{x-2}-\dfrac{x+1}{2-x}$

The denominators are negatives of each other.

$$\frac{2x+3}{x-2}-\frac{(x+1)(-1)}{(2-x)(-1)}=\frac{2x+3}{x-2}-\frac{(-x-1)}{(-2+x)}$$

$$=\frac{2x+3}{x-2}-\frac{(-x-1)}{x-2}$$

$$=\frac{(2x+3)-(-x-1)}{x-2}$$

$$=\frac{2x+3+x+1}{x-2}$$

$$=\frac{3x+4}{x-2}$$

19. Add or subtract, as indicated, and simplify the following.

(3) a. $\frac{5}{x+4}+\frac{3}{x+1}$

The L.C.D. is $(x+4)(x+1)$.

$$\frac{(5)(x+1)}{(x+4)(x+1)}+\frac{3(x+4)}{(x+1)(x+4)}=\frac{5x+5+3x+12}{(x+1)(x+4)}$$
$$=\frac{8x+17}{(x+1)(x+4)}$$

(3) b. $\frac{x+5}{x-4}-\frac{x+6}{x-3}$

The L.C.D. is $(x-4)(x-3)$.

$$\frac{(x+5)(x-3)}{(x-4)(x-3)}-\frac{(x+6)(x-4)}{(x-3)(x-4)}=\frac{(x+5)(x-3)-(x+6)(x-4)}{(x-4)(x-3)}$$
$$=\frac{\left(x^2+2x-15\right)-\left(x^2+2x-24\right)}{(x-4)(x-3)}$$
$$=\frac{x^2+2x-15-x^2-2x+24}{(x-4)(x-3)}$$
$$=\frac{9}{(x-4)(x-3)}$$

20. Add or subtract, as indicated, and simplify the following.

(4) a. $\dfrac{5}{a^2-16}+\dfrac{2}{a^2-4a}$

L.C.D. $= a(a+4)(a-4)$

$$\frac{5}{(a+4)(a-4)}+\frac{2}{a(a-4)}=\frac{(5)(a)}{(a+4)(a-4)(a)}+\frac{2(a+4)}{a(a+4)(a-4)}$$
$$=\frac{5a+2a+8}{a(a+4)(a-4)}$$
$$=\frac{7a+8}{a(a+4)(a-4)}$$

(4) b. $\dfrac{2x-3y}{x^2-y^2}-\dfrac{1}{x-y}$

L.C.D. $= (x+y)(x-y)$

$$\frac{2x-3y}{(x+y)(x-y)}-\frac{(1)(x+y)}{(x-y)(x+y)}=\frac{2x-3y-x-y}{(x-y)(x+y)}$$
$$=\frac{x-4y}{(x-y)(x+y)}$$

(4) c. $\dfrac{3x}{x^2-9}+\dfrac{2}{x+3}-\dfrac{1}{x-3}$

L.C.D. $= (x+3)(x-3)$

$$\frac{3x}{(x+3)(x-3)}+\frac{2}{x+3}-\frac{1}{x-3}=\frac{3x}{(x+3)(x-3)}+\frac{(2)(x-3)}{(x+3)(x-3)}-\frac{(1)(x+3)}{(x-3)(x+3)}$$
$$=\frac{3x+2x-6-x-3}{(x+3)(x-3)}$$
$$=\frac{4x-9}{(x+3)(x-3)}$$

(3) 21. Solve the following equation.

$$\frac{4x+1}{2x+3}=\frac{2x-1}{x+2}$$

L.C.D. $=(2x+3)(x+2)$

$x \neq \frac{-3}{2}$ and -2, or the nonpermissible values are $\frac{-3}{2}$ and -2.

$$\cancel{(2x+3)}^{1}(x+2)\left(\frac{4x+1}{\cancel{2x+3}_{1}}\right)=(2x+3)\cancel{(x+2)}^{1}\left(\frac{2x-1}{\cancel{x+2}_{1}}\right)$$

$$(x+2)(4x+1)=(2x+3)(2x-1)$$

$$4x^2+9x+2=4x^2+4x-3$$

$$9x+2=4x-3$$

$$5x+2=-3$$

$$5x=-5$$

$$x=-1$$

(3) 22. Solve the following equation.

$$\frac{2x-2}{x+2}=2+\frac{5-x}{x+2}$$

L.C.D. $=x+2$

$x \neq -2$, or the nonpermissible value is -2.

$$\cancel{(x+2)}^{1}\left(\frac{2x-2}{\cancel{x+2}_{1}}\right)=(x+2)(2)+\cancel{(x+2)}^{1}\left(\frac{5-x}{\cancel{x+2}_{1}}\right)$$

$$2x-2=(x+2)(2)+(5-x)$$

$$2x-2=2x+4+5-x$$

$$2x-2=x+9$$

$$x-2=9$$

$$x=11$$

(3) 23. Solve the following equation.

$$\frac{4}{x+2}-\frac{3}{x+3}=\frac{8}{x^2+5x+6}$$

$$\text{L.C.D.}=(x+2)(x+3)$$

$x \neq -2,\ -3$, or the nonpermissible values are -2 and -3.

$$\overset{1}{\cancel{(x+2)}}(x+3)\left(\frac{4}{\underset{1}{\cancel{x+2}}}\right)-(x+2)\overset{1}{\cancel{(x+3)}}\left(\frac{3}{\underset{1}{\cancel{x+3}}}\right)=\overset{1}{\cancel{(x+2)}}\overset{1}{\cancel{(x+3)}}\left(\frac{8}{\underset{1}{\cancel{(x+2)}}\underset{1}{\cancel{(x+3)}}}\right)$$

$$(x+3)(4)-(x+2)(3)=8$$
$$4x+12-3x-6=8$$
$$x+6=8$$
$$x=2$$

(3) 24. What number must be added to the numerator and the denominator of $\frac{1}{7}$ to give a fraction equal to $\frac{5}{6}$?

Let n = the number to be added.

$$\frac{1+n}{7+n}=\frac{5}{6}$$

$$\text{L.C.D.}=(6)(7+n)$$

$n \neq -7$, or the nonpermissible value is -7.

$$(6)\overset{1}{\cancel{(7+n)}}\left(\frac{1+n}{\underset{1}{\cancel{7+n}}}\right)=\overset{1}{\cancel{6}}(7+n)\left(\frac{5}{\underset{1}{\cancel{6}}}\right)$$

$$(6)(1+n)=(7+n)(5)$$
$$6+6n=35+5n$$
$$6+n=35$$
$$n=29$$

Twenty-nine has to be added to the numerator and the denominator.

(3) 25. Solve for x in the following equation.

$$\frac{x}{3}-\frac{x+5}{4}=\frac{2x+3}{6}$$

The L.C.D. is 12.

$$(\cancel{12}^{4})\left(\frac{x}{\cancel{3}_{1}}\right)-(\cancel{12}^{3})\left(\frac{x+5}{\cancel{4}_{1}}\right)=(\cancel{12}^{2})\left(\frac{2x+3}{\cancel{6}_{1}}\right)$$

$$4x-3(x+5)=2(2x+3)$$
$$4x-3x-15=4x+6$$
$$-3x-15=6$$
$$-3x=21$$
$$x=\frac{21}{-3}$$
$$x=-7$$

Topic 1

_____ marks

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